IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/MATSD/SP1/ENG/TZ0/XX/M+

MARKSCHEME

May 2006

MATHEMATICAL STUDIES

Standard Level

Paper 1

Paper 1 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

The number of marks for each question is 6.

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

1 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** answers (irrespective of working shown)

R Marks awarded for clear **Reasoning**

ft Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

(a) All marking must be done using a **red** pen.

(b) Marks must be noted on candidates' scripts as in the markscheme:

- A correct answer only needs ***C*** marks to be shown, otherwise show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*.
- Write down and circle the total for **each question** at the end of the question.
- Transfer the total for **each question** to the front cover sheet and write down the total mark for the paper.

(c) In this paper, the maximum mark is awarded for a **correct answer on the answer line**. **There is no need to check the working!** Award ***C*** marks and move on.

(d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.

(e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.

(f) Working crossed out by the candidate should not be awarded any marks.

(g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.

(h) If correct working results in a correct answer **in the working box** but then further working is developed, full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**.

Please note: Assignment of marks to the answers in all the following examples is for demonstration purposes only. Marks for actual examination questions will not necessarily follow the same pattern.

Implementation:
Question: Factorise $x^2 - 5x - 6$

Markscheme	Candidates' Scripts	Marking
$(x\text{-}6)(x+1)$ ***(A1)(A1)***	(i) Answer line: $(x\text{-}6)(x+1)$	***(C2)***
	(ii) Answer line: $(x+6)(x+1)$	***(A0) (A1)***
	(iii) Working box: $(x\text{-}6)(x+1)$ followed by answer line: x = 6 and -1, or just 6,-1	***(A1)*** ***(A0)***
	(iv) Working box: $(x\text{-}6)(x+1)$ then $x = 6,\text{-}1$ followed by answer line: x = 6 and -1, or just 6,-1 or factors and roots together	***(A1)*** ***(A0)***
	but	
	(v) Working box: $(x\text{-}6)(x+1)$ then $x = 6,\text{-}1$ followed by answer line: $(x - 6)\ (x+1)$ only	***(C2)***
	(vi) Working box: $(x\text{-}6)(x+1)$ then $x = 6,\text{-}1$ and answer line empty	***(A1)(A0)***

Question: Using Pythagoras to find a side of a triangle:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ ***(M1)(A1)*** (3.61 3sf)	(i) Answer line: $\sqrt{13}$ or 3.61 or both	***(C2)***
	(ii) Working box: $\sqrt{9+4} = \sqrt{13} = 6.50$ Answer line 6.5	***(M1)*** ***(A0)***
	(iii) Working box: $\sqrt{9+4} = \sqrt{13} = 6.50$ Answer line empty	***(M1)(A0)***
	(iv) Working box: $\sqrt{9+4} = \sqrt{13} = 3.61$ but answer line 3.16	***(M1)(A1)***
	For further considerations on this problem with regard to accuracy see later examples.	*(obvious transcription error)*

Question: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} = -\frac{6}{5}$ ***(M1)(A1)***	(i) Working: $m = \frac{9-3}{0-5} = -\frac{6}{5}$ followed by $y = -6x/5 + 9$ but -6/5 on answer line	***(C2)***
	(ii) Working box: $m = \frac{9-3}{0-5} = -\frac{6}{5}$ followed by $y = -6x/5 + 9$ and then answer line: either $y = -6x/5+9$ or $y = -6x/5$ or nothing at all on the answer line	***(M1)*** ***(A0)*** (even if -6/5 is **also** on the answer line)

3 Follow through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'** appended to the eligible mark(s).

- If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or wrong by large order of magnitude) then the final ***A*** mark should not be awarded. If in doubt, contact your team leader.

- If a question is transformed by an error into a **different**, **much simpler question** then follow through might not apply or might be reduced. In this situation consult your team leader and record the decision on the candidate's script.

- To award follow through marks for a question part, **there must be working present for that part** and not just an answer based on the follow through. An isolated follow through answer, with no working, must be regarded as incorrect and receives no marks **even if it seems approximately correct**.

- Inadvertent use of radians will be penalised the first time it occurs. Subsequent use, even in later questions, will normally be allowed follow through marks, unless the answer is unrealistic. Cases of this kind will be addressed on an individual basis.

Implementation: The following examples illustrate correct use of the **follow through** process in straightforward situations.

Question: An investment problem with two different rates of interest and a total amount of $600 split across the rates in consecutive periods:

Markscheme	Candidate's Script	Marking
(a) $ 600 × 1.02 ***(M1)*** = $ 612 ***(A1)*** (b) $\$ \left(\frac{612}{2} \times 1.02\right) + \left(\frac{612}{2} \times 1.04\right)$ ***(M1)*** = $ 630.36 ***(A1)*(ft)** ***Note: The (M1) is for splitting the value from (a) and forming a sum of products.*** Here the ***(ft)*** indicates a possible follow through from part (a).	Case (i) (a) Final amount after 1st period = $ 600 × 1.02 = $ 602 (b) Amount after 2nd period = 301 × 1.02 + 301 × 1.04 = $ 620.06 but note Case (ii) an ***(M0)*** almost always prohibits the **associated** ***(ft)*** so (a) $ 600 × 1.02 = $ 602 (b) $ 602 × 1.04 = $626.08 Case (iii) (a) $ 600 × 1.02 = $ 602 (b) No working. 620.06 on answer line.	 ***(M1)*** ***(A0)*** ***(M1)*** ***(A1)*(ft)** ***(M1)(A0)*** ***(M0)(A0)*(ft)** ***(M1)(A0)*** ***(M0)(A0)*(ft)**

Question: Finding angles and lengths using trigonometry

Markscheme	Candidate's Script	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)*** $A = 22.0^\circ$ ***(A1)*** (b) $x = 7 \tan A$ ***(M1)*** = 2.83 ***(A1)*(ft)**	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$ $A = 41.8^\circ$ (b) case (i) $x = 7 \tan A$ = 6.26 ***but*** case (ii) 6.26	***(M1) (A0) (use of sine rule but with wrong values)*** ***(A0)*** *(Note: the 2nd* ***(A1)*** *here was not marked* ***(ft)*** *and cannot be awarded because there was an earlier error in the same question part.)* ***(M1)*** ***(A1)*(ft)** ***(C0)*(ft)**

4 Using the Markscheme

This markscheme presents a particular way in which each question might be worked and how it should be marked.

(a) As ***A*** marks are normally **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark, unless otherwise instructed in the markscheme. (See the first example above). Similarly ***(A1)(R0)*** cannot be awarded for an answer which is accidentally correct for the wrong reasons given.

Implementation: Question: (a) χ^2 calculated followed by (b) degrees of freedom found and (c) and (d) comparison to critical value. (Interdependence of ***A*** and ***R*** marks.)

Markscheme	Candidate's Script	Marking
	Case (i)	
(a) $\chi_{calc}^2 = 3.92$ ***(A1)***	(a) $\chi_{calc}^2 = 3.92$	***(A1)***
(b) $n = 4$ ***(A1)***	(b) $n = 4$	***(A1)***
(c) $\chi_{crit}^2 = 9.488$ ***(A1)*(ft)**	(c) Don't know?	***(A0)***
(d) Do not reject null hypothesis ***(A1)*(ft)**	(d) Do not reject null hypothesis	***(A0)***
because $\chi_{calc}^2 < \chi_{crit}^2$ ***(R1)*(ft)**	because $\chi_{calc}^2 > 0$	***(R0)***
	Case (ii)	
	(a) $\chi_{calc}^2 = 3.92$	***(A1)***
	(b) $n = 4$	***(A1)***
	(c) $\chi_{crit}^2 = 4.488$	***(A0)***
	(d) Do not reject null hypothesis because $\chi_{calc}^2 < \chi_{crit}^2$	***(A1)*(ft)** ***(R1)*(ft)**
	Case (iii)	
	(a) $\chi_{calc}^2 = 3.92$	***(A1)***
	(b) $n = 1$	***(A0)***
	(c) $\chi_{crit}^2 = 3.841$	***(A1)*(ft)**
	(d) Reject null hypothesis because $\chi_{calc}^2 > \chi_{crit}^2$	***(A1)*(ft)** ***(R1)*(ft)**

(b) **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner that is consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*. This includes alternatives obtained with a graphic display calculator.

Example: Question to find the coordinates of a vertex of a given quadratic

Working	Marks
$f(x) = 2x^2 + 7x - 3$	
$x = -\frac{b}{2a} = -\frac{7}{4}$	***(M1)(A1)***
(M1) *for use of –b/2a,* ***(A1)*** *for correct answer*	
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$	***(M1)(A1)*(ft)**
(M1) *for using f(-7/4),* ***(A1)*** *for answer.*	
Coordinates are (-7/4,-73/8)	***(A1)*(ft)**
OR	**OR**
$f'(x) = 4x + 7$, $4x+7 = 0$	***(M1)***
so $x = -7/4$	***(A1)***
(M1) *for attempting to take a derivative and setting it to 0*	
(A1) *for answer*	
$f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$	***(M1)(A1)*(ft)**
(M1) *for using f(-7/4),* ***(A1)*** *for answer.*	
Coordinates are (-7/4,-73/8)	***(A1)*(ft)**

(c) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$. On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

(d) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; $1 \cdot 7$; 1,7 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement):
A'; $\bar{A}$; A^c; $U - A$; $(A$ U/A.

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\bar{p}$; $\sim p$.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

(e) Discretionary ***(d)*** marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, ***(d)*** should be used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to 3 significant figures.
A penalty known as an **ACCURACY PENALTY *(AP)*** is applied if an answer is either

(i) rounded incorrectly to 3 significant figures or

(ii) rounded correctly or incorrectly to some other level of accuracy.

This penalty is applied to the **final answer** of a question part only. It applies **also** when an exact answer is incorrectly rounded.

THE ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER! Subsequent accuracy errors can be **ignored** and full marks awarded if all else is correct.

An accuracy penalty must be recorded in proximity to the incorrect answer as ***(A0)(AP)***.
Examiners must record the occurrence of an accuracy penalty by writing ***(AP)*** next to the relevant question total on the front of the cover sheet.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. This is **NOT** an accuracy penalty. A mark for specified accuracy can be regarded as a ***(ft)*** mark regardless of an immediately preceding ***(M0)***.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. If the rounding is incorrect, an accuracy penalty should be applied as detailed above. Exact answers such as $\frac{1}{4}$ can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of π and answers taking the form of square roots of integers **(even if exact squares)** or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}, \sqrt{9}$) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.
Answers **with no supporting working** (usually from a GDC), which are written correct to more than 3 significant figures can be awarded full marks with an ***(AP)*** then applied. When this happens, multiple C marks can be split (*e.g.* ***(A1)(A0)(AP)*** or ***(C1)(C0)(AP)***. **Unsupported** answers with less than 3 significant figures must be deemed incorrect even if they seem approximately correct.

An accuracy penalty should not be applied to an answer that is already incorrect for some other reason.

Special cases

Answers involving units of currency can be accepted correct to 3 significant figures or correct to the nearest currency unit (*e.g.* dollar) or correct to the nearest hundredth unit (*e.g.* cent). Allow all these cases to follow through to later question parts.

An answer taken directly from the IB chi squared statistical table can be given and used to the same level of accuracy as appears in the table (3 decimal places) or correct to 3 significant figures.

For judging equivalence between 3sf and use of minutes and seconds for angles, guidelines have been issued to paper setters. This problem will be dealt with on an individual basis as the need arises.

Examples: The Pythagoras example used before:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4}=\sqrt{13}$ ***(M1)(A1)*** (3.61 3sf)	(i) Working box: nothing but answer line: 3.6 or 4	***(C0)***
	(ii) Working box: nothing but answer line: 3.60555	***(C1)(C0)(AP)***
	(iii) Working box: $\sqrt{9+4}=\sqrt{13}$ Answer line: 3.6	***(M1)*** ***(A0)(AP)***
	(iv) Working box: $\sqrt{9+4}=\sqrt{13}$ Answer line: 3.60555	***(M1)*** ***(A0)(AP)***
	(v) Working box: $\sqrt{9+4}=\sqrt{13}=3.60$	***(M1)(A0)(AP)***
	(vi) Working box: $\sqrt{9+4}=\sqrt{14}=3.74$ transferred, or not, to answer line	***(M1)(A0)***

If the question specified *e.g.* correct to 4 decimal places for the answer, then there would be one extra mark available as follows:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4}=\sqrt{13}$ ***(M1)(A1)*** $= 3.6056$ (4dp) ***(A1)*****(ft)**	(i) Working box: nothing but answer line: 3.606	***(C0)***
	(ii) Working box: nothing but answer line: 3.6055	***(C0)***
	(iii) Working box: $\sqrt{9+4}=\sqrt{13}$ Answer line 3.6	***(M1)(A1)*** ***(A0)***
	(iv) Working box: $\sqrt{9+4}=\sqrt{13}$ Answer line: 3.60555	***(M1)(A1)*** ***(A0)***
	(v) Working box: $\sqrt{9+4}=\sqrt{14}$ $= 3.7417$ whether transferred to answer line or not.	***(M1)(A0)*** ***(A1)*****(ft)**
	(vi) Working box: $\sqrt{9-4}=\sqrt{5}$ $= 2.2361$ whether transferred to answer line or not.	***(M0)(A0)*** ***(A1)*****(ft)** *Note: this is a special case, where the initial* ***(M0)*** *does not determine the final* ***(A0)***
	(vii) Answer line: 3.61 or 3.606 wrong answers, no working.	***(C0)***

Premature Rounding

Accuracy errors in a final answer, which result from premature rounding earlier in the same question part, should not receive an accuracy penalty. There are two situations.
If there is a mark available for a prematurely rounded answer and the rounding occurs at this stage, then the inappropriate rounding should be penalised with ***(A0)*** but the answer can then be allowed to follow through to the end of the question. If the first stage of the answer is correct but rounded further on, then it should be penalised at an appropriate place close to where it is rounded. Some discretion should be used to deny a **(ft)** mark if the rounding is very bad and the answer far from its required value.

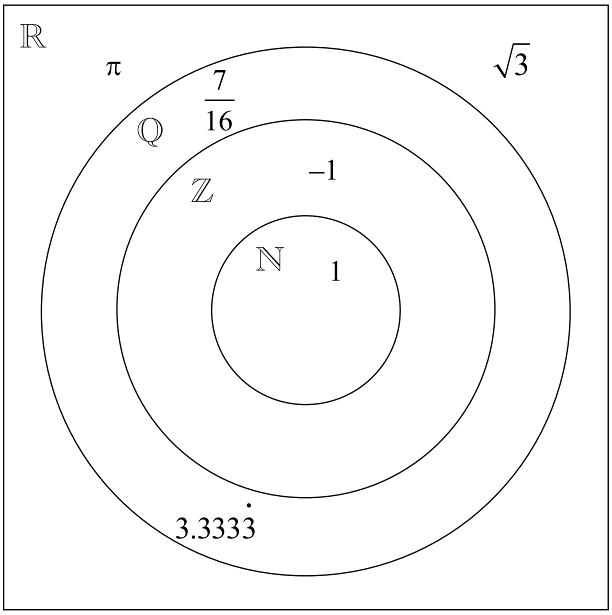
Example: Question: sine rule used to find angle A, with angle B and side b known but side a is first calculated using Pythagoras in an adjoining triangle.

Markscheme	Candidate's Script	Marking
$a = \sqrt{25+36} = \sqrt{61}$ ***(M1)(A1)***	(i) $a = \sqrt{25+36} = \sqrt{61}$ $= 7.8$	***(M1)(A0)***
$\dfrac{\sin(A)}{\sqrt{61}} = \dfrac{\sin(32)}{5}$ ***(M1)(A1)*(ft)**	$\dfrac{\sin(A)}{7.8} = \dfrac{\sin(32)}{5}$	***(M1)(A1)*(ft)**
$A = 55.9^\circ$ ***(A1)*(ft)**	$A = 55.8^\circ$	***(A1)*(ft)**
	(ii) $a = \sqrt{25+36} = \sqrt{61}$	***(M1)(A1)***
	$\dfrac{\sin(A)}{7.8} = \dfrac{\sin(32)}{5}$	***(M1)(A0)***
	$\text{A} = 55.8^\circ$	***(A1)*(ft)**
	(iii) $a = \sqrt{25+36} = \sqrt{61}$	***(M1)(A1)***
	$\dfrac{\sin(A)}{7.8} = \dfrac{\sin(32)}{5}$	***(M1)(A0)***
	$A = \sin^{-1}(0.83) = 56.1^\circ$	***(A0)***
	(iv) $a = \sqrt{25+36} = \sqrt{61} = 8$	***(M1)(A0)***
	$\dfrac{\sin(A)}{8} = \dfrac{\sin(32)}{5}$	***(M1)(A1)*(ft)**
	$A = 58.0^\circ$	***(A0)*(ft)** *(The rounding is severe and the answer quite far from correct).*

6 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation.

Q1



(A1) (A1)
(A1)
(A1)
(A1)
(A1)
(C6)

*For any number entered exactly once, in the correct position, award **(A1)**, if incorrect award **(A0)**.*

*If all numbers entered in all regions award **(A0)**.*

If any number is entered in more than one region, penalise that number as follows:

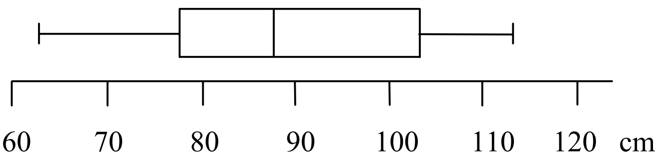
*(i) If none of the regions is correct award **(A0)***

*(ii) If one of the regions is correct but other appearances of that number are in the COMPLEMENT of the correct set, award **(A0)** the first time this is seen.*

*(iii) If one of the regions is correct but other appearances of that number are in a SUBSET of the correct set award **(A0)** the first time this is seen.*

Apply each of (ii) and (iii) at most once and award ft *marks when the error is seen repeatedly, however, (ii) and (iii) may not both be applied to the same number and if both these errors are present with more than one number involved, follow through cannot be used until both penalties have been applied.*

[6 marks]

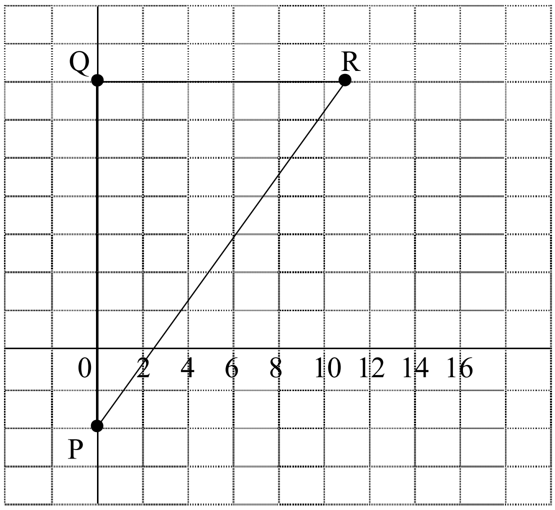
Q2	(a) 19 seedlings	***(A1)***	***(C1)***
	(b) (i) median 88 cm	***(A1)***	
	(ii) 1^{st} quartile 78 cm, 3^{rd} quartile 103 cm (both correct)	***(A1)***	***(C2)***
	(c) $112 - 63 = 49$ cm	***(A1)***	***(C1)***
	Accept 63 and 112 both seen, if they appear in the answer space for (c) or under working for (c) (but not just implied or written on the box plot).		
	(d) 		
	Box with correct median and quartiles marked. *Both correct whiskers joined to box with straight lines* *Allow maximum errors of* ± 2. *Perfectly ruled lines are not essential.*	***(A1)*** **(ft)** ***(A1)*** **(ft)**	***(C2)***
			[6 marks]

Q3	(a) 1.265×10^{-1} *Exact answer, i.e* ***not*** *to 3 s.f. is required for first* ***(A1)****.* *Can* **(ft)** *the power from wrong magnitude coefficient.*	***(A1)(A1)*****(ft)**	***(C2)***
	(b) $0.13 \quad (1.3 \times 10^{-1})$	***(A1)*****(ft)**	***(C1)***
	(c) $\dfrac{0.13-0.1265}{0.1265} \times 100\,\%$ $= 2.77\,\%$ *Award* ***(M1)*** *for substituting numbers into the formula,* ***(A1)*** *for substitution of the correct numbers. The answer must be a percentage for award of the second* ***(A1)****.* *In this session, allow the formula with absolute value taken and 100 missing, with answer 0.0277. Note: this instruction is designed to address the problem that an old, incorrect version of the formula booklet was used by some schools.*	***(M1)(A1)*****(ft)** ***(A1)*****(ft)**	***(C3)***
			[6 marks]
Q4	(a) If I am wearing my hat (then) the sun is not shining. ***(A1)*** *for if with or without then* ***(A1)*** *for both correct statements with implication in correct direction* *Use of 'therefore' is incorrect.*	***(A1)(A1)***	***(C2)***
	(b) see table below ***(A1)*** for $\neg p$ column, ***(A1)*****(ft)** for $q \Rightarrow \neg p$ column	***(A1)(A1)*****(ft)**	***(C2)***
	(c) $\neg p \Rightarrow q$ (or $q \Leftarrow \neg p$) *Do not split the* ***(A2)*** *Accept only these two statements.*	***(A2)***	***(C2)***
			[6 marks]

Q4 (b)

p	q	$\neg p$	$q \Rightarrow \neg p$
T	T	**F**	**F**
T	F	**F**	**T**
F	T	**T**	**T**
F	F	**T**	**T**
		(A1)	***(A1)*****(ft)**

Q5	(a) mean $= \dfrac{(8\times150+16\times160+11\times170+7\times180+3\times190)}{45}$	***(M1)(M1)***	
	*Award **(M1)** for five correct products shown or implied in the numerator, **(M1)** for denominator 45.* = \$ 165.78 per week *(allow \$ 166)*	***(A1)***	***(C3)***
	For $165.\dot{7}$ *or* $165\frac{7}{9}$ *award **(C3)** for exact answer.* *For 165.77 award **(C2)** and no **(AP)**.* *For 165.77 with no working award **(C2)(A0)(AP)**.*		
	(b) \$ 164.99 *(\$ 165)*	***(A1)***	***(C1)***
	(c) $16\times\$\,155 = \$\,2480$ and $16\times164.99 = \$\,2639.84$ (\$ 2640)	***(M1)***	
	*The **(M1)** is for a sensible attempt to calculate both bounds or for showing division by 16 of any of the values (i) to (v).*		
	\$ 2430 is not possible	***(A1)*(ft)**	***(C2)***
	Follow through if wrong modal group is used in (b).		
			[6 marks]

Q6			
	(a) Point R on diagram	***(A1)***	***(C1)***
	(b) angle QPR $= \tan^{-1}\left(\frac{11}{20}\right)$	***(M1)***	
	*Award **(M1)** for tan or arctan seen.*		
	$= 28.8^{\circ}$ $(28^{\circ}49')$	***(A1)*****(ft)**	***(C2)***
	(c) Area $= \frac{1}{2}ab\sin C$		
	$= \frac{1}{2}\times 20\times 9\times \sin 28.8$	***(M1)(A1)*****(ft)**	
	$= 43.4$	***(A1)*****(ft)**	
	***M1** for correct area formula with substitutions, **(A1)** for correct substitutions.*		***(C3)***
			[6 marks]

Q7	(a) $Q(0) = 25$	***(A1)***	***(C1)***
	(b) $Q(20) = 0.003(20)^2 - 0.625(20) + 25 = 13.7$	***(A1)***	***(C1)***
	(c) $Q(20) = 13.7$ units and $Q(10) = 19.05$ difference = 19.05 – 13.7 *(allow* 13.7 – 19.05*)* = 5.35	***(M1)***	
	average = $\frac{5.35}{10}$ = 0.535 units per minute (sign must be positive) *The wording may have confused some candidates. Hence allow* $\frac{19.05+13.7}{2} = 16.375$ (16.4) ***(M1)(A1)(C2)*** *and for answer 16.3 (which is Q(15)) award just **(C1)**.* *If the interval is divided into multiple parts (e.g 10) then the value averaged over these, award no marks.*	***(A1)*(ft)**	***(C2)***
	(d) $0.003t^2 - 0.625t + 25 = 0$ *A sketch showing the first root can earn the **(M1)**.* Energy runs out after 54.0 minutes *(accept 54 but no mark for 154)* *53.99 receives **(A0)(AP)**, 53 is wrong, for 53-54 award **(A1)***	***(M1)*** ***(A1)***	***(C2)*** ***[6 marks]***

Q8	(a) $\left(\frac{8}{15}\times\frac{7}{14}\right)$	***(M1)***	
	$=\frac{56}{210}=\frac{4}{15}$ (0.267)	***(A1)***	***(C2)***
	(M1) *is for a product including at least one correct fraction.*		
	(b) $\left(\frac{4}{15}\times\frac{3}{14}\right)+\left(\frac{3}{15}\times\frac{2}{14}\right)$	***(M1)(M1)***	
	(M1) *is for adding two products, the other* ***(M1)*** *is if both products attempt to deal with non-replacement and the numbers are not ridiculous.*		
	$=\frac{18}{210}$ or $\frac{3}{35}$ (0.0857)	***(A1)***	***(C3)***
	If one correct product is doubled this receives ***(M1)(M0)(A0)***		
	(c) The probability is 0. *(Allow answer 'impossible' or equivalent.)*	***(A1)***	***(C1)***
			[6 marks]
Q9	(a) $\pi R^2 = 36\pi$ so $R = 6$ cm	***(M1)(A1)***	***(C2)***
	(b) Use cosine rule. $AB^2 = 6^2 + 6^2 - 2\times 6\times 6\cos(110^\circ)$	***(M1)(A1)*****(ft)**	
	$AB^2 = 96.6$ $AB = 9.83$ cm	***(A1)*****(ft)**	
	OR	**OR**	
	$\frac{6}{\sin(35^\circ)}=\frac{AB}{\sin(110^\circ)}$	***(M1)(A1)*****(ft)**	
	AB = 9.83 cm	***(A1)*****(ft)**	
	OR	**OR**	
	$\frac{110}{2}=55$		
	$\sin(55^\circ)=\frac{\frac{1}{2}AB}{6}$	***(M1)(A1)*****(ft)**	
	$AB = 9.83$	***(A1)*****(ft)**	***(C3)***
	If this method is used, then the $\frac{1}{2}$ *AB must be evident to obtain the* ***(M1)*** *and the first* ***(A1)*** *requires the 55 and the 6 to be correct.*		
	(c) $L = \sqrt{36\pi}$ or $6\sqrt{\pi}$ or 10.6 cm	***(A1)***	***(C1)***
			[6 marks]

Q10	(a) 800×0.766 $= \$\,612.80$ ($\$\,613$) *(Accept \$612.8 as an exact answer.)*	*(M1)(A1)*	*(C2)*
	(b) $\dfrac{612.80}{0.785}$ $= \$\,780.64$ ($\$\,781$ or \$780.89 if following from 613)	*(M1)* *(A1)*(ft)	*(C2)*
	If the wrong rate or the wrong process are chosen in part (a), then follow through to parts (b) and (c) using the alternative rate or process.		
	(c) $800 - 780.64 = \$\,19.36$ $\dfrac{19.36}{800} \times 100\% = 2.42\,\%$	*(A1)*(ft) *(A1)*(ft)	*(C2)*
	780.89 follows through to 2.39% and 781 follows to 2.38%.		***[6 marks]***
Q11	(a) $6C + 3V = 163.17$ $9C + 2V = 200.53$	*(A1)* *(A1)*	*(C2)*
	*If **both** addition signs missing, award **(A0)(A1)**(ft)*		
	(b) *GDC use is expected.* Solve simultaneously to find $V = \$\,17.69$ (\$17.7)	*(M1)(A1)*(ft)	*(C2)*
	*\$ 18.35 here receives **(A0)***		
	*A reasonable attempt to solve on paper without the GDC can receive **(M1).***		
	(c) $9 \times 18.35 = 165.15$ $180 - 165.15$ $= \$14.85$ (\$14.9)	*(M1)* *(A1)*(ft)	*(C2)*
	If C and V are reversed in (b) and (c) all the marks can be treated as **(ft)** *in (c), however if the same wrong answer for C appears in both (b) and (c) then (c) can receive at most **(M1)(A0)**. In the former case the answers are \$ 159.21 and \$ 20.79 respectively.*		***[6 marks]***

Q12 (a)

cubic function $f(x)$	graph label
$f(x) = x^3 + a$	B
$f(x) = (x-a)^3 + a$	E
$f(x) = x^3$	C
$f(x) = (x-a)^3$	D

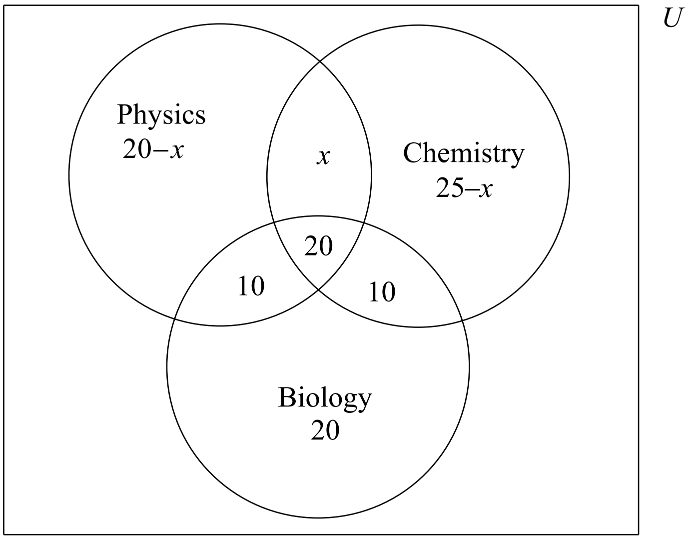
(A1)
(A1)
(A1)
(A1) ***(C4)***

*If the same letter is written in every box, award **(A0) not (A1)**.*

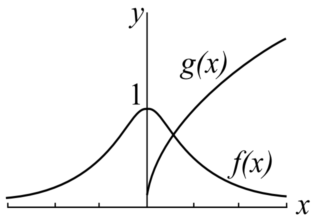
(b) Graph A. ***(A1)*** ***(C1)***

(c) Two of them. *(Allow 'D and C').* ***(A1)*** ***(C1)***

[6 marks]

Q13	The full diagram is as follows, but need not be fully filled in.  		
	(a) $50-(30+x)=20-x$	***(M1)(A1)***	***(C2)***
	*Award **(C2)** for <u>either</u> of these as final answer.*		
	*Can award **(M1)** for 50 minus a sum of numbers intended to count the entries in **all** the Physics intersections. Also award **(M1)(A0)** if the sign is distributed incorrectly, giving answers such as 50-30+x or 20+x.*		
	(b) $60+x+(20-x)+(25-x)=90$	***(M1)(M1)***	
	*Award **(M1)** for 90 seen on one side of the equation, **(M1)** for a reasonable attempt to order the entries.*		
	Therefore $x=15$.	***(A1)*(ft)**	***(C3)***
	(c) Number studying at least two subjects is 55. *Allow correct* **(ft)** *with no marking shown*	***(A1)*(ft)**	***(C1)*** ***[6 marks]***

Q14	(a) $f'(x) = 12x^3 - \frac{2}{x^2} - \frac{1}{4}$	***(A1)(A1)(A1)***	
	Award one mark for each correct term (with no other constant) $f''(x) = 36x^2 + \frac{4}{x^3}$ ***(M1)*** *is for differentiating the first derivative.* ***(A1)*** *is for the correct answer.* *(Can allow* **(ft)** *here if first derivative is wrong, as long as the second differentiation is not greatly simplified, e.g. a 2nd derivative of just* $36x^2$ *should only receive* ***(M1)(A0)*** **(ft)**. *If 2nd derivative* ***only*** *is written, and is not entirely correct, award* ***(A2)*** *for each correct term.*	***(M1)(A1)*****(ft)**	***(C5)***
	(b) $f''(1) = 40$ **(ft)** *for any 2nd derivative in (a).*	***(A1)*****(ft)**	***(C1)***
			[6 marks]

Q15	(a)	With the given domain, the correct answer is 	*(A1)* *(A1)*(ft) *(A1)*(ft)	*(C3)*
		Award ***(A1)*** *for a neat window complying reasonably with the requirements,* *The window must clearly have used x values from –3 to 3 and y values at least from 0 to 1. Axis labels are not essential. Some indication of scale must be present but this need not be a formal scale, e.g. tick marks, a single number on each axis or coordinates of the intersection are all adequate.* *Award* ***(A1)*** *for each curve correct and correctly labelled with f and g or the expressions for f and g. Can follow through both curves, for example if curves are incomplete due to a poor window, and penalise only once if both curve labels are missing. Examiners should familiarise themselves with the graph of* $\frac{1}{x^2}+1$ *as this is expected to appear in error. With the correct window, this graph will not be seen at all, but with a larger y interval it might look a little like the correct graph except that it would have asymptotes at x = 0 and y = 1. Award* ***(A0)*** *for this curve.*		
	(b)	One solution.	*(A1)*(ft)	*(C1)*
	(c)	Solution occurs at the point of intersection of the curves, where $x = 0.569840$ 0.570.	*(M1)(A1)*	*(C2)*
		The ***(M1)*** *can also be awarded for the intersection point indicated on the sketch.* *(0.57 is an* ***(AP)****)* *If a coordinate pair is given as the answer and the x value is correct with no method mentioned, award* ***(C1)*** *or if the method* **is** *mentioned, award* ***(M1)(A0).*** *Can follow through if curve* $\frac{1}{x^2}+1$ *is drawn, answer to (c) is then 1.75.*		***[6 marks]***